Probability and Applied Statistics

Formula Sheet

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# 1 What Is Statistics?

## Definition 1.1: Mean

The *mean* of a sample of measured responses is given by

The corresponding population mean is denoted .

## Definition 1.2: Variance

The *variance* of a sample of measurements is the sum of the square of the differences between the measurements and their mean, divided by . Symbolically, the sample variance is

The corresponding population variance is denoted by the symbol .

## Definition 1.3: Standard Deviation

The *standard deviation* of a sample of measurements is the positive square root of the variance; that is,

The corresponding population standard deviation is denoted by .

# 2 Probability

## Extra 2.1: Union

The union of and , denoted by , is the set of all points in or or both.

## Extra 2.2: Intersection

The intersection of and , denoted by or by , is the set of all points in both and .

## Extra 2.3: Complement

If is a subset of , then the complement of , denoted by , is the set of points that are in but not in . Note that .

## Extra 2.4: Mutually Exclusive

Two sets, and , are said to be *disjoint*, or *mutually exclusive*, if . That is, mutually exclusive sets have no points in common. Note that, for any set , and are mutually exclusive.

## Extra 2.5: Distributive Laws

## Extra 2.6: De Morgan’s Laws

## Definition 2.1: Experiment

An *experiment* is the process by which an observation is made.

## Definition 2.2: Simple Event

A *simple event* is an event that CANNOT be decomposed. Each simple event corresponds to one and only one *sample point*. The letter *E* with a subscript will be used to denote a simple event or the corresponding sample point. Note: An event that CAN be decomposed into other events is called a *compound event*.

## Definition 2.3: Sample Space

The *sample space* associated with an experiment is the set consisting of all possible sample points. A sample space will be denoted by .

## Definition 2.4: Discrete Sample Space

A *discrete sample space* is one that contains either a finite or a countable number of distinct sample points.

## Definition 2.5: Event

An *event* in a discrete sample space is a collection of sample point­s—that is, any subset of *S*.

## Definition 2.6: Probability

Suppose is a sample space associated with an experiment. To every event in ( is a subset of ), we assign a number, , called the *probability* of , so that the following axioms hold:

Axiom 1:

Axiom 2:

Axiom 3: If form a sequence of pairwise mutually exclusive events in S (that is, if ), then

## Theorem 2.1: Rule

With elements and elements , it is possible to form pairs containing one element from each group.

## Definition 2.7: Permutation

An ordered arrangement of distinct objects is called a *permutation*. The number of ways of ordering distinct objects taken at a time will be designated by the symbol .

## Theorem 2.2: Number of Distinct Arrangements

## Theorem 2.3: Multinomial Coefficient

The number of ways of partitioning distinct objects into distinct groups containing objects, respectively, where each object appears in exactly one group and , is

## Definition 2.8: Combination

The number of *combinations* of objects taken at a time is the number of subsets, each of size , that can be formed from the objects. This number will be denoted by or .

## Theorem 2.4: Number of Unordered Subsets

The number of unordered subsets of size *r* chosen (without replacement) from *n* available objects is

## Definition 2.9: Conditional Probability

The *conditional probability* of an event , given that an event has occurred, is equal to

provided . [The symbol is read “probability of given .”]

## Definition 2.10: Independent Events

Two events and are said to be *independent* if any one of the following holds:

Otherwise, the events are said to be *dependent*.

## Theorem 2.5: The Multiplicative Laws of Probability

The probability of the intersection of two events and is

If and are independent, then

## Theorem 2.6: The Additive Law of Probability

The probability of the union of two events and is

If and are mutually exclusive events, and

## Theorem 2.7: Complement Rule

If is an event, then

## Definition 2.11: Partition

For some positive integer , let the sets be such that

1.

2. , for

Then the collection of sets is said to be a *partition* of .

## Theorem 2.8: The Law of Total Probability

Assume that is a partition of such that , for . Then for any event

## Theorem 2.9: Bayes’ Rule

Assume that is a partition of such that , for . Then

## Definition 2.12: Random Variable

A *random variable* is a real-valued function for which the domain is a sample space.

## Definition 2.13: Random Sample

Let and represent the numbers of elements in the population and sample, respectively. If the sampling is conducted in such a way that each of the samples has an equal probability of being selected, the sampling is said to be random, and the result is said to be a *random sample*.

# 3 Discrete Random Variables and Their Probability Distributions

## Definition 3.1: Discrete

A random variable is said to be *discrete* if it can assume only a finite or countably infinite number of distinct values.

## Definition 3.2: Probability Function for Y

The probability that takes on the value , , is defined as the *sum of the probabilities of all sample points in* that are assigned the value . We will sometimes denote by .

## Definition 3.3: Probability Distribution

The *probability distribution* for a discrete variable can be represented by a formula, a table, or a graph that provides for all .

## Theorem 3.1: Properties of Discrete Probability Distribution

For any discrete probability distribution, the following must be true:

1. , for all .

2. , where the summation is over all values of with nonzero probability.

## Definition 3.4: Expected Value of a Discrete Random Variable

Let be a discrete random variable with the probability function . Then the *expected value* of , , is defined to be

## Theorem 3.2: Expected Value of a Real-Valued Function of a Discrete Random Variable

Let be a discrete random variable with probability function and be a real-valued function of . Then the expected value of is given by

## Definition 3.5: Standard Deviation of a Discrete Random Variable

If is a random variable with mean , the variance of a random variable is defined to be the expected value of . That is,

The *standard deviation* of is the positive square root of .

## Theorem 3.3: Expected Value of Nonrandom Quantity

Let be a discrete random variable with probability function and be a constant. Then .

## Theorem 3.4: Expected Value of the Product of a Constant and a Function of a Random Variable

Let be a discrete random variable with probability function , be a function of , and be a constant. Then

## Theorem 3.5: Expected Value of a Sum of Functions of a Random Variable

Let be a discrete random variable with probability function and be functions of . Then

## Theorem 3.6: Variance of a Discrete Random Variable

Let be a discrete random variable with probability function and mean ; then

## Definition 3.6: Binomial Experiment

A *binomial experiment* possesses the following properties:

1. The experiment consists of a fixed number, , or identical trials.

2. Each trial results in one of two outcomes: success, , or failure, .

3. The probability of success on a single trial is equal to some value and remains the same from trial to trial. The probability of a failure is equal to .

4. The trials are independent.

5. The random variable of interest is , the number of successes observed during the trials.

## Definition 3.7: Binomial Distribution

A random variable is said to have a *binomial distribution* based on trials with success probability if and only if

and

## Theorem 3.7: Mean and Variance of a Binomial Random Variable

Let be a binomial random variable based on trials and success probability . Then

and

## Definition 3.8: Geometric Probability Distribution

A random variable is said to have a *geometric probability distribution* if and only if

## Theorem 3.8: Mean of a Geometric Random Variable

If is a random variable with a geometric distribution,

and